

HOMEWORK 1

Que: (30 p) Cauchy inequality is given by:

$$\left| \sum_{i=1}^n a_i b_i \right|^2 \leq \sum_{i=1}^n |a_i|^2 \sum_{i=1}^n |b_i|^2.$$

where $a_i, b_i, 1 < i < n$ are complex numbers. Prove Cauchy inequality by induction.

Que: (20 p) Prove that

$$\left| \frac{a-b}{1-\bar{a}b} \right| < 1$$

if $|a| < 1$ and $|b| < 1$.

Que: (20 p)

4. Show that there are complex numbers z satisfying

$$|z-a| + |z+a| = 2|c|$$

if and only if $|a| \leq |c|$. If this condition is fulfilled, what are the smallest and largest values of $|z|$?

Que: (30 p)

Express the following complex numbers in the polar form.

a) $4 + 3i$

b) $2 - 5i$

c) $-2 - 2i$

d) $-1 + 4i$

e) $\frac{\sqrt{5}}{2+2i}$

f) $\frac{4i}{3+6i}$

g) $\frac{1+i}{i-1}$

h) $(\sqrt{3} + i)^2$

Express the following complex numbers in the Cartesian form such as $a + ib$.

a) $\sqrt{2}e^{i\pi/4}$

b) $\sqrt{5}e^{-i\pi/3}$

c) $4e^{i\pi}$

d) $2e^i$

Calculate following complex numbers.

a) $(2 + i)^2$

b) $(3i - 1)^3$

c) $(4i + 5)^2(-3i - 1)^4$

d) $(i - 1)^5(1 - 2i)^6$

e) $\frac{(1-4i)^2(3-3i)}{(-5i-2)(1+i)^4}$

f) $(3 - 2i)^8(-1 - i)^4$

Find all values of the following roots.

a) $(3i)^{1/2}$

b) $(-i)^{1/4}$

c) $(-1)^{1/4}$

d) $27^{1/6}$

e) $(-4)^{1/2}$

f) $(64i)^{1/3}$

g) $(1 + i)^{1/2}$

h) $(-1)^{1/12}$

For the complex number $z = 2 - i$ calculate following functions.

a) z^n

b) $\frac{1}{z}$

c) z^{-n}

d) $z^3 + 2z^2 + 5z + 4 - 2i$

e) $|z|^2$

f) $(\bar{z})^2$

g) $z \cdot \bar{z}$

h) $\text{Arg}(z - 2) + \text{Arg}(z + i)$